Math 10A HW 5 Solutions

(1) (a) $f(x) = e^x$, $a = 0$

$$T_2(x) = f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2$$

$$\begin{aligned} f(x) &= e^x & & f(0) = 1 \\ f'(x) &= e^x & \longrightarrow\ & f'(0) = 1 \\ f''(x) &= e^x & & f''(0) = 1 \end{aligned}$$

$\Rightarrow T_2(x) = 1 + x + \frac{1}{2}x^2$

(b) $g(x) = e^{x^2}$, $a = 0$

$$T_2(x) = g(0) + \frac{g'(0)}{1!}(x-0) + \frac{g''(0)}{2!}(x-0)^2$$

$$\begin{aligned} g(x) &= e^{x^2} & \longrightarrow\ & g(0) = 1 \\ g'(x) &= e^{x^2} \cdot 2x & \longrightarrow\ & g'(0) = 0 \\ g''(x) &= e^{x^2} \cdot 2 + 2x(e^{x^2} \cdot 2x) & \longrightarrow\ & g''(0) = 2 \\ &= 2e^{x^2} + 4x^2 e^{x^2} & & \end{aligned}$$

$\Rightarrow T_2(x) = 1 + x^2$

(2) $f(x) = \ln(1+x)$, $a = 0$

$T_1(x) = f(0) + f'(0)(x-0) = x$

$f(x) = \ln(1+x) \longrightarrow f(0) = 0$
$f'(x) = 1/1+x \longrightarrow f'(0) = 1$
$f''(x) = -1(1+x)^{-2} \longrightarrow f''(0) = -1$
$= -1/(1+x)^2$

$T_2(x) = f(0) + f'(0) \cdot x + \dfrac{f''(0)}{2!} \cdot x^2$

$\Rightarrow T_2(x) = x - \dfrac{x^2}{2}$

So for $x = 0.01$ we have $T_1(0.01) = 0.01$
$T_2(0.01) = 0.00995$

(3) True!

$T_2(x) = f(a) + f'(a)(x-a) + \dfrac{f''(a)(x-a)^2}{2}$

$T_2(x) = f(a) + f'(a)x - af'(a) + \dfrac{x^2 f''(a)}{2}$

$- axf''(a) + \dfrac{a^2 f''(a)}{2}$

$\Rightarrow T_2'(x) = f'(a) + xf''(a) - af''(a)$

$T_2'(a) = f'(a)$!

(4) False! Let $f(x) = e^{x}$.

(5) $g(x) = x^5 + 2 \quad , \quad x_0 = 1$

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = 1 - \frac{g(1)}{g'(1)} = 1 - \frac{3}{5} = \frac{2}{5}$$

(6) $g(x) = x + \cos(\pi x) \quad , \quad x_0 = \frac{1}{2}$

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = \frac{1}{2} - \frac{g(\frac{1}{2})}{g'(\frac{1}{2})} = \frac{1}{2} - \frac{(\frac{1}{2})}{1 - \pi \sin(\pi/2)}$$

$$\Rightarrow x_1 = \frac{1}{2} - \frac{\frac{1}{2}}{1-\pi} = \frac{1}{2} - \frac{1}{2(1-\pi)} = \frac{1-\pi}{2(1-\pi)} - \frac{1}{2(1-\pi)}$$

$$\Rightarrow x_1 = \frac{-\pi}{2(1-\pi)}$$

(7) Let $f(x) = x^2 - 5$ (since $x^2 - 5 = 0 \Leftrightarrow x = \sqrt{5}$)
and let $x_0 = 2$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{(x_0^2 - 5)}{2x_0} = 2 - \frac{(4-5)}{4} = 2 + \frac{1}{4}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 9/4 - \frac{f(9/4)}{f'(9/4)} = 9/4 - \frac{1/16}{9/2} \approx 2.236$$

(8) False! Consider the function $f(x) = \sqrt[3]{x}$ with initial $x_0 \neq 0$.

(9) False! We should have $f(x) = x^2 - 17$ since $x^2 - 17 = 0 \Leftrightarrow x = \sqrt{17}$.